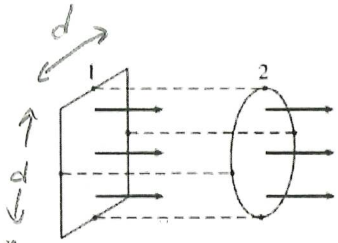
1.

The diameter of the circle equals the edge length of the square. They are in a uniform electric field. Is the electric flux Φ_1 through the square larger than, smaller than, or equal to the electric flux Φ_2 through the circle? Explain.

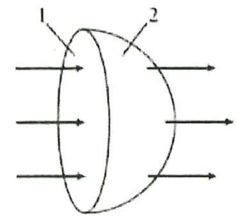


For flat surface & uniform $\vec{E}$, $\Phi_e = \int \vec{E} \cdot d\vec{A} = \int E\,dA = E\int dA = EA$

$\Phi_1 = Ed^2$

$\Phi_2 = E\pi\left(\frac{d}{2}\right)^2 = E\frac{\pi}{4}d^2 \qquad \frac{\pi}{4} < 1 \quad \therefore \Phi_2 < \Phi_1$

Is the electric flux Φ_1 through the circle larger than, smaller than, or equal to the electric flux Φ_2 through the hemisphere? Explain.

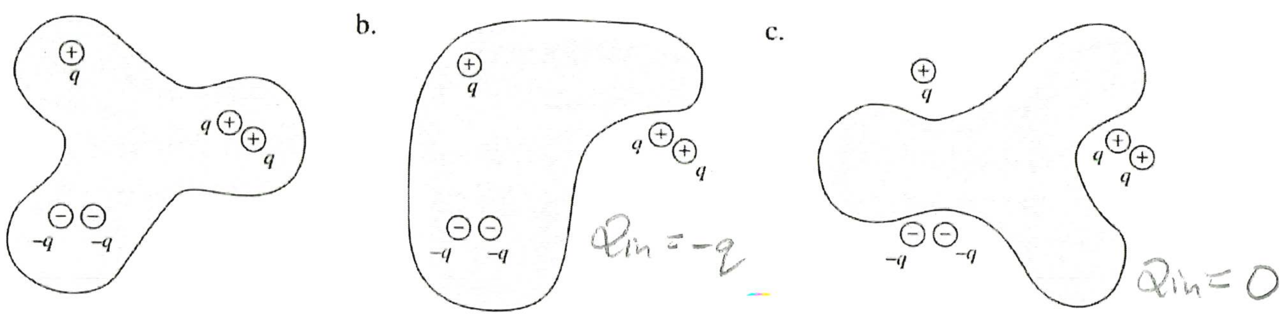


Same no. of $\vec{E}$-field lines cross each surface.

$\therefore |\Phi_1| = |\Phi_2|$

2.

What is the electric flux through each of these closed surfaces? (The charges in the gray areas are inside the closed surfaces.) Give your answers as multiples of q/ϵ_0.



a. $Q_{in} = q$ $\qquad \Phi_e = q/\varepsilon_0$

b. $Q_{in} = -q$ $\qquad \Phi_e = -q/\varepsilon_0$

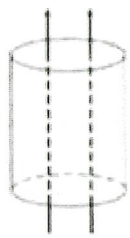
c. $Q_{in} = 0$ $\qquad \Phi_e = 0$

$\Phi_e = \frac{Q_{in}}{\varepsilon_0}$

3.

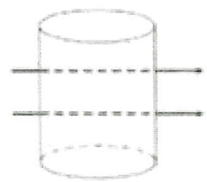
For each of the closed cylinders shown below, are the electric fluxes through the top, the wall, and the bottom positive (+), negative (−), or zero (0)? Is the net flux positive, negative, or zero?

a.



$\Phi_{top} =$ +
$\Phi_{wall} =$ 0
$\Phi_{bot} =$ −
$\Phi_{net} =$ 0

b.



$\Phi_{top} =$ 0
$\Phi_{wall} =$ 0
$\Phi_{bot} =$ 0
$\Phi_{net} =$ 0

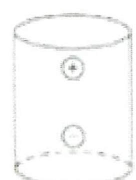
c.

$\Phi_{top} =$ −
$\Phi_{wall} =$ −
$\Phi_{bot} =$ −
$\Phi_{net} =$ −

d.

$\Phi_{top} =$ −
$\Phi_{wall} =$ +
$\Phi_{bot} =$ +
$\Phi_{net} =$ 0

e.

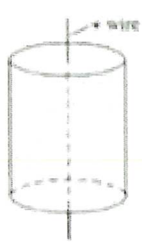


$\Phi_{top} =$ +
$\Phi_{wall} =$ 0
$\Phi_{bot} =$ −
$\Phi_{net} =$ 0

f.

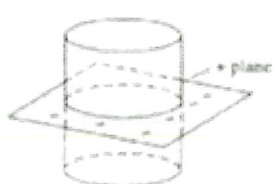
$\Phi_{top} =$ −
$\Phi_{wall} =$ 0
$\Phi_{bot} =$ +
$\Phi_{net} =$ 0

g.



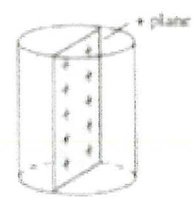
$\Phi_{top} =$ 0
$\Phi_{wall} =$ +
$\Phi_{bot} =$ 0
$\Phi_{net} =$ +

h.



$\Phi_{top} =$ +
$\Phi_{wall} =$ 0
$\Phi_{bot} =$ +
$\Phi_{net} =$ +

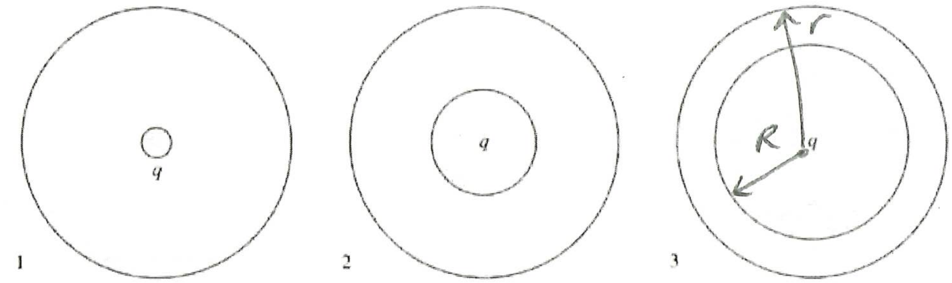
i.



$\Phi_{top} =$ 0
$\Phi_{wall} =$ +
$\Phi_{bot} =$ 0
$\Phi_{net} =$ +

4.

Three charges, all the same charge q, are surrounded by three spheres of equal radii.



a. Rank in order, from largest to smallest, the fluxes Φ_1, Φ_2, and Φ_3 through the spheres.

Order:
Explanation: $Q_{in} = q$ ∀ three. $\Phi_e = \frac{Q_{in}}{\varepsilon_0}$ $\therefore \Phi_1 = \Phi_2 = \Phi_3$

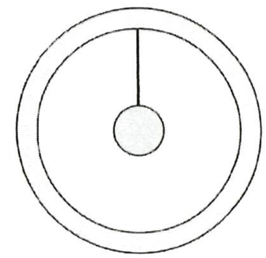
b. Rank in order, from largest to smallest, the electric field strengths E_1, E_2, and E_3 on the surfaces of the spheres.

Order:
Explanation: For a spherical charge at position $r > R$, $\vec{E}$ is like that due to pt. charge.

In all three cases E at r is $E = \frac{q}{4\pi\varepsilon_0 r^2}$

5.

A small metal sphere hangs by a thread within a larger, hollow conducting sphere. A charged rod is used to transfer positive charge to the outer surface of the hollow sphere.



a. Suppose the thread is an insulator. After the charged rod touches the outer sphere and is removed, are the following surfaces positive, negative, or not charged?

The small sphere: not charged

The inner surface of the hollow sphere: not charged

The outer surface of the hollow sphere: +

b. Suppose the thread is a conductor. After the charged rod touches the outer sphere and is removed, are the following surfaces positive, negative, or not charged?

The small sphere: not charged

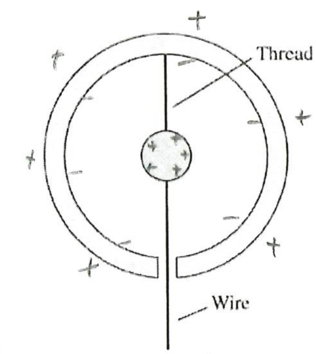
The inner surface of the hollow sphere: not charged

The outer surface of the hollow sphere: +

All charge at outer surface of conductor!

6.

A small metal sphere hangs by an insulating thread within a larger, hollow conducting sphere. A conducting wire extends from the small sphere through, but not touching, a small hole in the hollow sphere. A charged rod is used to transfer positive charge to the wire. After the charged rod has touched the wire and been removed, are the following surfaces positive, negative, or not charged?



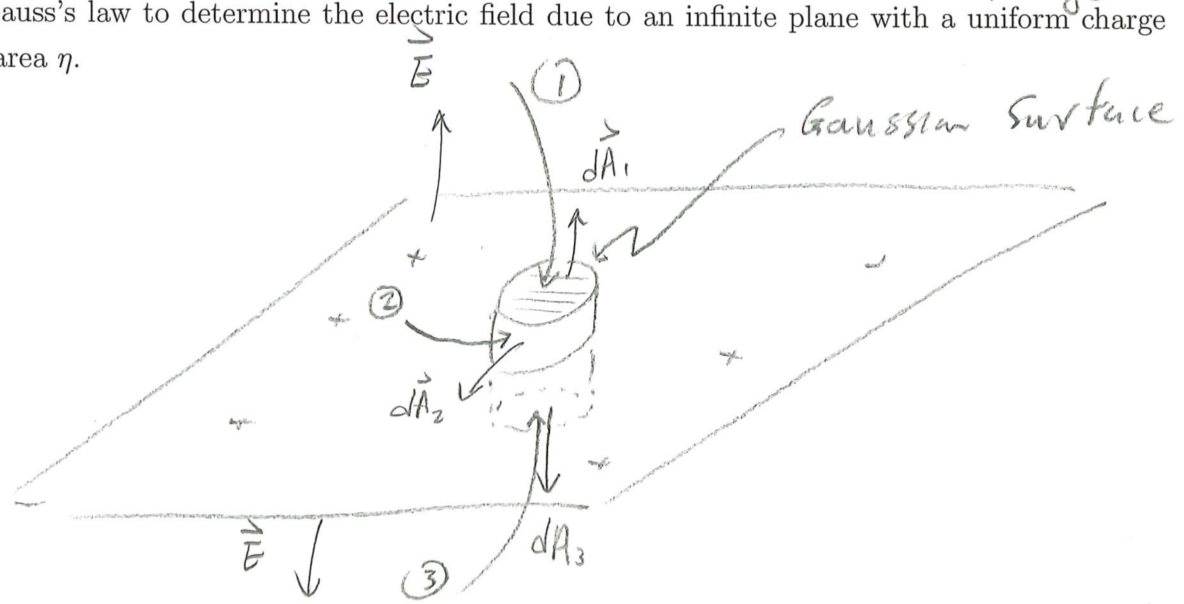
The small sphere: +

The inner surface of the hollow sphere: −

The outer surface of the hollow sphere: +

outer conductor is overall neutral. ∴ pos. outer surface charge balances neg. inner surface charge.

7. Use Gauss's law to determine the electric field due to an infinite plane with a uniform charge per unit area η.



$$\oint \vec{E}\cdot d\vec{A} = \int_{①} \vec{E}\cdot d\vec{A}_1 + \int_{②} \vec{E}\cdot d\vec{A}_2 + \int_{③} \vec{E}\cdot d\vec{A}_3$$

$$\vec{E}\cdot d\vec{A}_1 = E\,dA_1$$
$$\vec{E}\cdot d\vec{A}_2 = 0$$
$$\vec{E}\cdot d\vec{A}_3 = E\,dA_3$$

$$\therefore \oint \vec{E}\cdot d\vec{A} = \int E\,dA_1 + \int E\,dA_3$$
$$= E\int dA_1 + E\int dA_3$$
$$= EA + EA = 2EA$$

A is area of top/btm of cylinder.

$$Q_{in} = \eta A$$

$$\therefore \oint \vec{E}\, d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

$$\Downarrow$$

$$2E\cancel{A} = \frac{\eta \cancel{A}}{\varepsilon_0}$$

$$\therefore \boxed{E = \frac{\eta}{2\varepsilon_0}}$$